

1. The degree of precision of the following quadrature rule is 1:

$$\int_{-1}^1 f(x) dx \approx 2f(0).$$

2. The closed Newton–Cotes rule with  $n$  points is exact for all polynomials of degree up to  $n$ .  
 3. The trapezium approximation is always wrong if  $f$  is not a polynomial. That is to say, for all non-polynomial function  $f$ , it holds that

$$\int_{-1}^1 f(x) dx \neq f(-1) + f(1).$$

4. The degree of precision of the following quadrature rule is 1:

$$\int_{-1}^1 f(x) dx \approx f(1).$$

5. Suppose that  $f \in C^\infty[a, b]$  and let  $I_n[f]$  denote the approximate integral of  $f$  using the composite trapezium rule with  $n$  integration points. Then it holds that

$$\lim_{n \rightarrow \infty} |I[f] - I_n[f]| = 0, \quad I[f] := \int_a^b f(x) dx.$$

6. Suppose that  $f \in C^\infty[a, b]$  and let  $I_n[f]$  denote the approximate integral of  $f$  using the composite trapezium rule with  $n$  integration points. Then there exists  $C$  such that

$$\forall n \in \{2, 3, \dots\}, \quad |I[f] - I_n[f]| \leq \frac{C}{n^2}.$$

7. Suppose that  $f \in C^\infty[a, b]$  and let  $I_n[f]$  denote the approximate integral of  $f$  using the composite trapezium rule with  $n$  integration points. Then it holds that

$$\lim_{n \rightarrow \infty} n^2 |I[f] - I_n[f]| = 0.$$

8. There exist  $w_1$  and  $w_2$  such that the degree of precision of the following rule is 2:

$$\int_{-1}^1 f(x) dx = w_1 f(0) + w_2 f(1).$$

9. In Julia, the following code implements the composite trapezium rule with  $n + 1$  points:

```
function I_approx(a, b, n)
    x = LinRange(a, b, n + 1)
    h = (b - a) / n
    return h*sum(f, x[1:n] .+ h/2)
end
```

10. In Julia, the following code implements the composite Simpson rule with  $n + 1$  points:

```
function I_approx(n)
    x = LinRange(a, b, n + 1)
    h = (b - a) / n
    result = 0.
    result += h/3 * f(x[1]) + h/3 * f(x[end])
    result += 4h/3 * sum(f, x[2:2:end-1])
    result += 2h/3 * sum(f, x[3:2:end-2])
    return result
end
```