

1. The degree of precision of the following quadrature rule is 1:

$$\int_{-1}^1 f(x) dx \approx 2f(0) - f(-1) + f(1).$$

2. The closed Newton–Cotes rule with  $n$  points is exact for all polynomials of degree up to  $n + 1$ .  
 3. Simpson’s composite integration rule is always correct if  $f$  is a polynomial.  
 4. The degree of precision of the following quadrature rule is 0:

$$\int_{-1}^1 f(x) dx \approx 2f(1).$$

5. Suppose that  $f \in C^\infty[a, b]$  and let  $I_n[f]$  denote the approximate integral of  $f$  using the composite Simpson rule with  $n$  integration points. Then it holds that

$$\lim_{n \rightarrow \infty} n |I[f] - I_n[f]| = 0, \quad I[f] := \int_a^b f(x) dx.$$

6. Suppose that  $f \in C^\infty[a, b]$  and let  $I_n[f]$  denote the approximate integral of  $f$  using the composite trapezium rule with  $n$  integration points. Then there exists  $C$  such that

$$\forall n \in \{2, 3, \dots\}, \quad |I[f] - I_n[f]| \leq \frac{C}{n^2}.$$

7. Suppose that  $f \in C^\infty[a, b]$  and let  $I_n[f]$  denote the approximate integral of  $f$  using the composite trapezium rule with  $n$  integration points. Then it holds that

$$\lim_{n \rightarrow \infty} n |I[f] - I_n[f]| = 0.$$

8. There exist  $w_1$  and  $w_2$  such that the degree of precision of the following rule is 4:

$$\int_{-1}^1 f(x) dx = w_1 f(0) + w_2 f(1) + w_3 f(-1) + w_4 f\left(\frac{1}{3}\right).$$

9. In Julia, the following code implements the composite trapezium rule with  $n$  points:

```
f(x) = sin(x)
function I_approx(a, b, n)
    x = LinRange(a, b, n)
    h = x[2] - x[1]
    return h/2 * sum(f, x[1:n]) + h/2 * sum(f, x[2:end])
end
```

10. In Julia, the following code implements the composite Simpson rule with  $n$  points:

```
f(x) = sin(x)
function I_approx(a, b, n)
    x = LinRange(a, b, n)
    h = x[2] - x[1]
    w = [1; [2 + 2(i%2) for i in 0:n-1]; 1]
    return h/3 * w'f.(x)
end
```