

1. The computational cost of the forward substitution algorithm, for a matrix of size $n \times n$, scales as $\mathcal{O}(n)$.
2. The computational cost to calculate the LU decomposition of an invertible matrix scales as $\mathcal{O}(n^2)$.
3. Suppose that \mathbf{A} is a symmetric, positive definite matrix. Calculating the Cholesky decomposition of \mathbf{A} requires fewer floating point operations than calculating the LU decomposition of \mathbf{A} .
4. The condition number $\kappa_2(\mathbf{A})$ of a square invertible matrix \mathbf{A} is always strictly larger than 1.
5. The spectral radius of a matrix square \mathbf{A} is zero if and only if \mathbf{A} is zero.
6. If the condition number of a matrix \mathbf{A} is very large ($\gg 1$), then it is possible that rounding errors arising from floating point arithmetic will have a large impact on the accuracy of the numerical solution to the linear system $\mathbf{Ax} = \mathbf{b}$ (calculated by LU decomposition followed by forward and backward substitution, for example).
7. Suppose that $\mathbf{A} \in \mathbf{R}^{n \times n}$ is symmetric and positive definite, and let $\mathbf{b} \in \mathbf{R}^n$. Consider the following iterative method for solving the linear system $\mathbf{Ax} = \mathbf{b}$:

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \omega (\mathbf{b} - \mathbf{Ax}^{(k)}). \quad (1)$$

This iteration converges to the exact solution of the linear system for all $\omega \in \mathbf{R}$.

8. The convergence speed of the iteration (1), for the optimal value of ω , is independent of the condition number $\kappa_2(\mathbf{A})$.
9. If \mathbf{A} is symmetric positive definite, there always exists $\omega \in \mathbf{R}$ such that the iteration (1) converges.
10. If \mathbf{A} is a nonzero matrix, then its norm $\|\mathbf{A}\|_2$ is strictly positive.
11. Suppose that we want to solve the linear systems $\mathbf{Ax} = \mathbf{b}$ and $\mathbf{Ax} = \mathbf{c}$ by a direct method. To this end, the LU decomposition of \mathbf{A} can be calculated only once.
12. Assuming that the matrix \mathbf{A} and the vector \mathbf{b} are already defined, write (on paper) Julia code implementing 100 iterations of (1).